Engineering non-equilibrium quantum phase transitions via causally gapped Hamiltonians

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Introduction

We introduce a phenomenological theory for many-body control of critical phenomena by engineering causally-induced gaps for quantum Hamiltonian systems [1]. The core mechanisms are controlling information flow within and between clusters that are created near a quantum critical point. To this end, we construct inhomogeneous quantum phase transitions via designing spatiotemporal quantum fluctuations. We show how the non-equilibrium evolution of disordered quantum systems can create new effective correlation length scales and effective dynamical critical exponents.

Quantum annealing

Hamiltonian for a system of N spins under a inhomogeneous driver field can be written as:

$$H(t) = -\sum_{n=1}^{N} g(n,t)\sigma_n^x - \sum_{<n,m>}^{N} J_{nm}\sigma_n^z\sigma_m^z. \quad (1)$$

It takes easy to prepare ground state at $g(n,t=0) \gg 1$, and within the given time-window of τ_Q quenches the transverse field to the final $g(n,t=\tau_Q)=0$. The goal is to sample from low-energy subspace of the problem Hamiltonian $H_p = -\sum_{<n,m>}^{N} J_{nm}\sigma_n^z\sigma_m^z$.

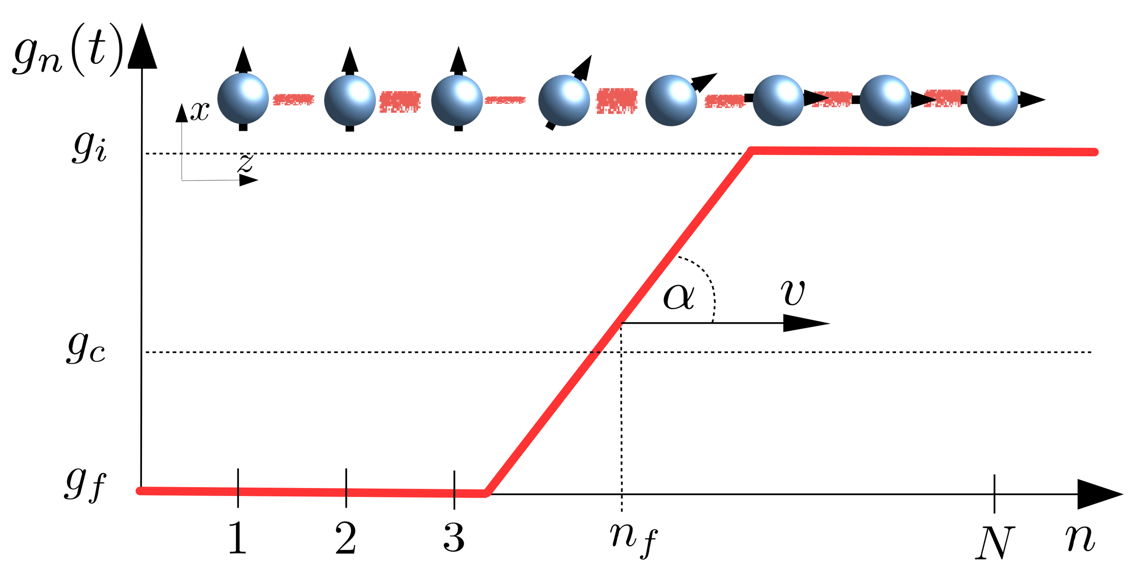
The difficulty is related to the presence of quantum critical points for intermediate values of g. They hamper the adiabaticity of the evolution, which otherwise would trace the instantaneous ground states between the initial one and the desired solution of H_p.

Inhomogeneous driving – single front

Here, we consider one-dimensional random Ising model, where $H_P = \sum_n J_{n,n+1}\sigma_n^z\sigma_{n+1}^z$. It has a critical point in the infinite-disorder universality class. The energy gap at the critical point vanishes as a stretched-exponential with the system size [2],

$$\Delta_{min} \sim e^{-CN^{1/2}}, \quad (2)$$

which means that exponentially long times are needed to keep improving the quality of the solution below a certain threshold. That can be overcome by driving a disordered spin chain by an inhomogeneous field [3],



Under inhomogeneous driving, the critical front is reached locally as the magnetic field is swept through the chain at velocity v. The length scale in which the external field is modulated, controlled by α, sets the number of spins in the neighborhood of the critical point.

By employing Kibble-Zurek like reasoning one can show that effective size of the critical region scales as [4],

$$\hat{\xi}^i \sim \alpha^{-\frac{\nu}{\nu+1}}, \quad (3)$$

for smooth $\alpha \ll 1$. It is set not by the size of the system N, but by the shape of the front α. As a result, the minimal gap is smoother out as [3, 1]

$$\hat{\Delta}^i_{min} \sim e^{-\tilde{C}\alpha^{-1/3}}. \quad (4)$$

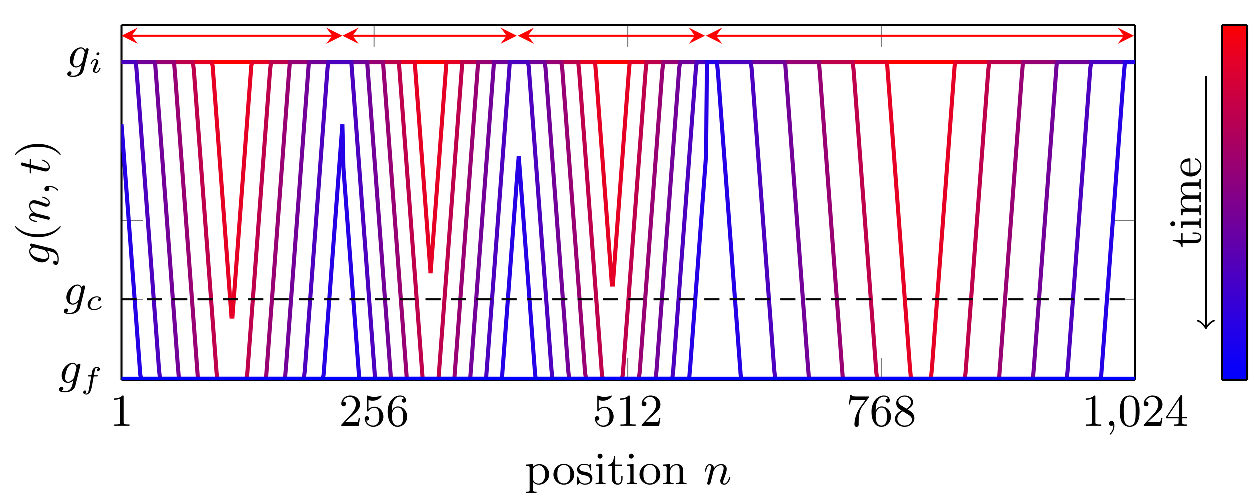
Combining those two scales we get the threshold velocity of the front

$$v_t \sim \hat{\Delta}^i_{min}\hat{\xi}^i \sim e^{-\tilde{C}\alpha^{-1/3}}. \quad (5)$$

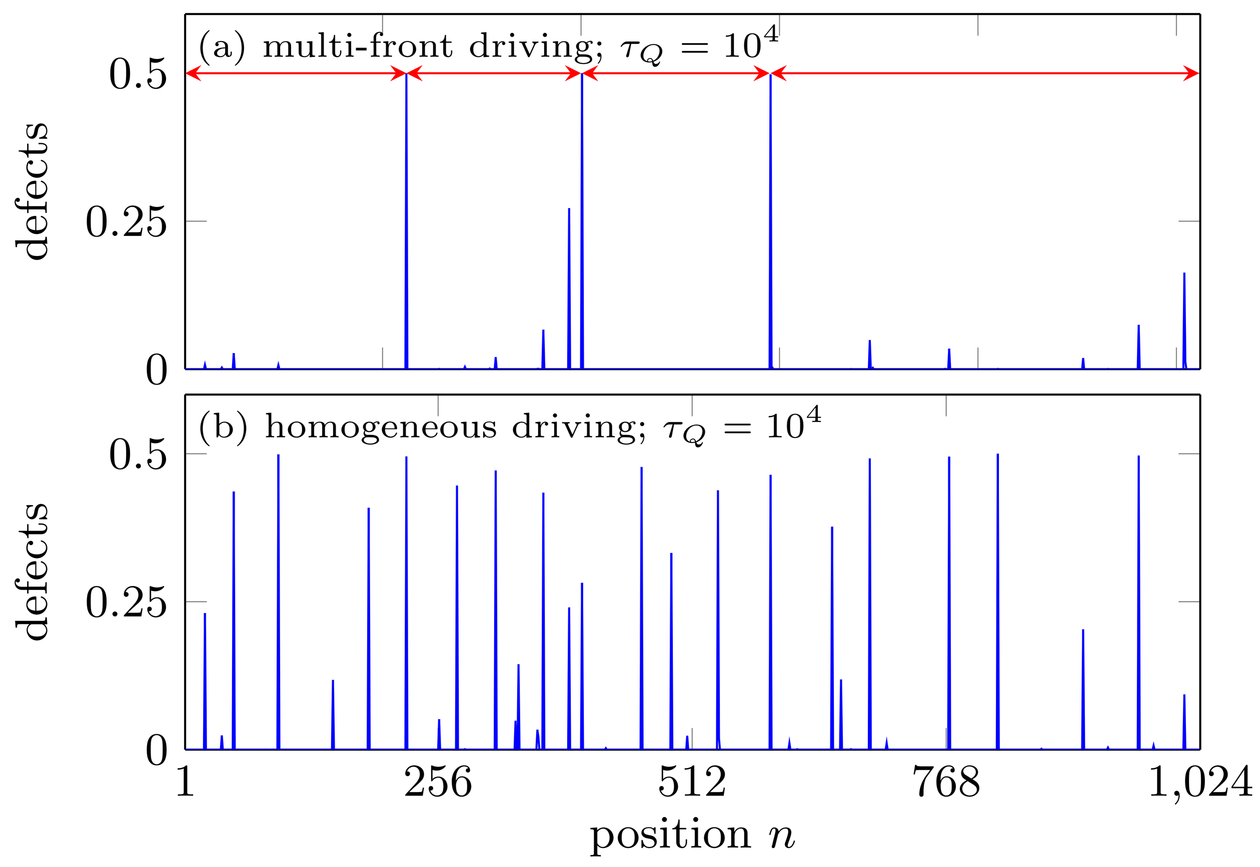
Driving the front with spatial velocity $v < v^t$ greatly reduces the number of defects generated during the quench.

Multiple fronts

We introduce a general family of protocols based on multiple fronts.



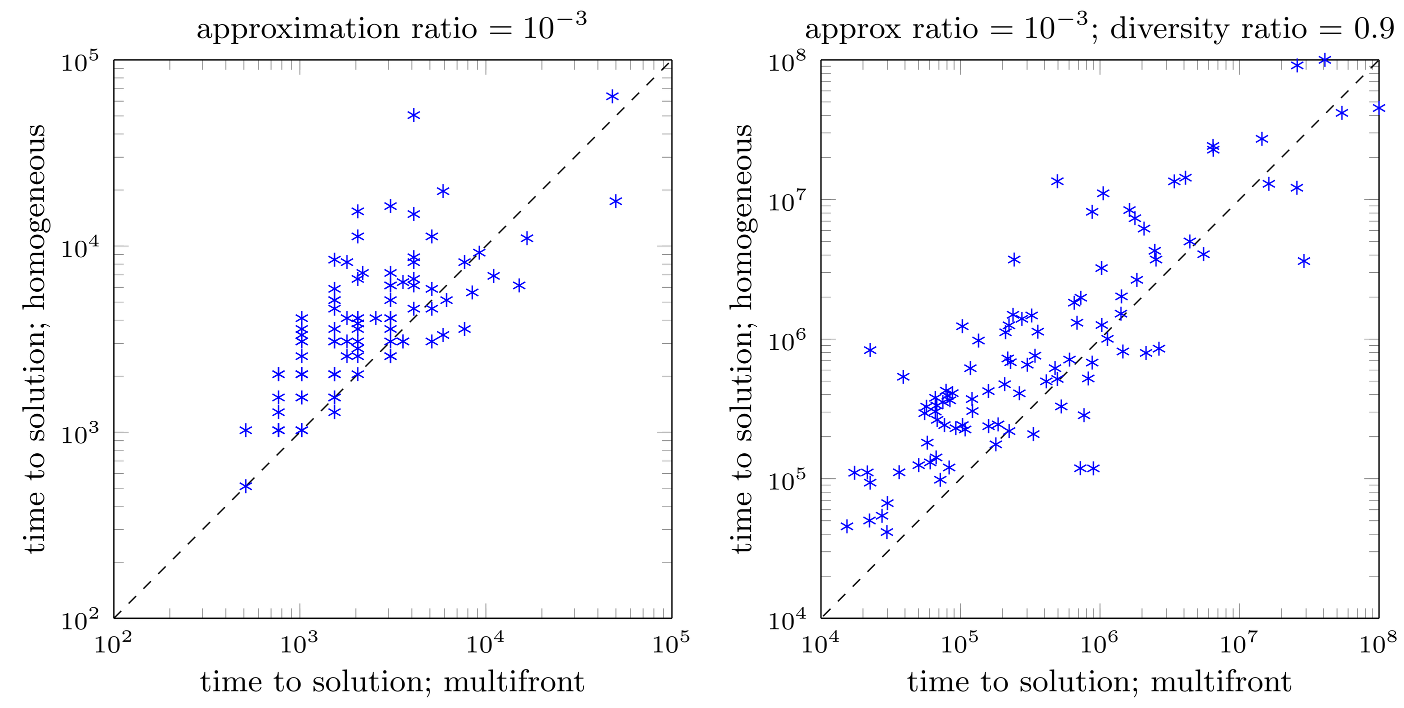
Each front is independently driving part of the system through the critical point, sharply suppressing the creation of the topological defects in the clusters. Here we show protocol with 4 clusters, where the borders between clusters are tuned to coincide with the weakest couplings $J_{n,n+1}$.



Distribution of topological defects for different protocols applied to a single instance of disorder Ising model. In (a), an inhomogeneous strategy essentially brushes the defects and place them at the borders between clusters where the energy penalties are minimal. As each cluster can break the symmetry independently, there is 50% probability of obtaining a defect between them. In (b), the defects are spread through the chain, and their density is expected to diminish as $\log^{-2}(\tau_Q)$.

Multiple fronts for frustrated quasi-1D

Quasi-1d chain with nonzero $J_{n,n+r}$ for $r \leq 3$, $N = 256$ spins.



We show time to solution to obtain given approximation ratio (small enough residual energy). We define *diversity ratio*, which quantifies what percentage of *significantly different* low energy solutions was found. Multi-front strategy shows speed-up according to both measures [5].

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References

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